Optimizing Tesla's charging station placement: A Game Theory Approach

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1 Problem Description

1.1 Situation

As the number of Tesla electric car users increases, Tesla needs to consider adding charging stations. Tesla gives the option of building charging stations to each parking lot in the city. Each parking lot can choose to build or not. However, building a charging station would only be profitable if none of the neighboring parking lot build a charging station themselves, since neighboring charging station need to share customers. Now imagine a number of parking lots within a certain region, connected by a road network, every parking lot is rational and selfish, and wishes to maximize their own utility..

1.2 Formulation

We can now formulate this as a game theory problem: We are given a connected graph G with at least 2 vertices. The set of parking lots (we say players) is the set of vertices of G. Each vertex v has two pure strategies: $\{Y, N\}$ (Yes or No). If v plays Y, then its utility is 1 if all its neighbors play N; otherwise its utility is -1. If v plays N, then its utility is 0 regardless of what its neighbors play.

1.3 Existence of Pure NE

Proof. Suppose this game has a pure Nash equilibrium, we claim that there's no player plays Y with utility -1, since the players can always choose to play N to increase their utilities from -1 to 0. We will use some concepts of graph theory in this question.

- A **dominating set** is a set of vertices D in a graph, such that any vertex of G is in D, or has a neighbor in D.
- An **independent set** is a set of vertices in a graph, no two of which are adjacent.
- An **independent dominating set** is a set of vertices in a graph, which has both property of dominating set and independent set.

We claim that every connected graph G has at least one independent dominating set. It can be proved by induction on the number of vertex in G

- Base case: when graph G only contains a single vertex, the vertex itself is an independent dominating set. Hence, statement hold in base case.
- Inductive hypothesis: assume, the graph G with $n-1, n \geq 1$ vertices contains an independent dominating set.
- Inductive step: let H be the graph with n vertices, pick an arbitrary vertex v, the subgraph H/v is a graph with $n-1$ vertices, and as the hypothesis, it contains an independent dominating set. Back to H, if v doesn't adjacent to any vertex in the independent dominating set, then add v into the independent dominating set, otherwise, do nothing. It leads to the result that H has an independent dominating set too.
- Conclusion: therefore, we can conclude that any connected graph G has at least one independent dominating set.

Let $ID \subseteq V(G)$ be an independent dominating vertex set in graph G. Let all vertex in ID choose strategy Y, and all the rest vertex choose strategy N. Since it follows the rule of independent dominating set, no adjacent vertex will choose Y together, therefore, all the player who choose Y will have utility 1. This is a pure Nash equilibrium already, since all player who choose Y have already maximized their utility, and all the player who choose N can't maximize their utility anymore, since if they choose Y, there must be another player that in adjacent vertex chosen Y already, which will change the player's utility into -1.
Therefore, this game has a pure Nash equilibrium indeed. □

2 Algorithm on finding Pure NE

2.1 Special Case: Cycle Graph

We claim that there's at least one player choose Y in any Nash equilibrium, since if all players choose N, then all of them would have incentive to change their strategy into Y to maximize their utility.

- When G is an odd cycle. Denote the vertices of G: $v_1, v_2, \cdots, v_{2n+1}$, where $n \geq 1$. Let v_i's strategy be Y. We follow this algorithm to construct the Nash equilibrium:

  1. Let v_n, v_m be neighbors of v_i. Since v_i chose Y, v_n, v_m must choose N.
  2. Let v_x be neighbor of v_m, v_y be neighbor of v_n. If v_x, v_y are not adjacent, then both of them could choose Y, then perform step 3. Otherwise, only one of them could choose Y, the other one must choose N, algorithm ends.
  3. Let v_p be neighbor of v_x, v_q be neighbor of v_y. Since v_x, v_y both chose Y, v_p, v_q should both chose N. If v_p, v_q are adjacent, algorithm ends. Otherwise, we let $v_m = v_p, v_n = v_q$, perform step 2.

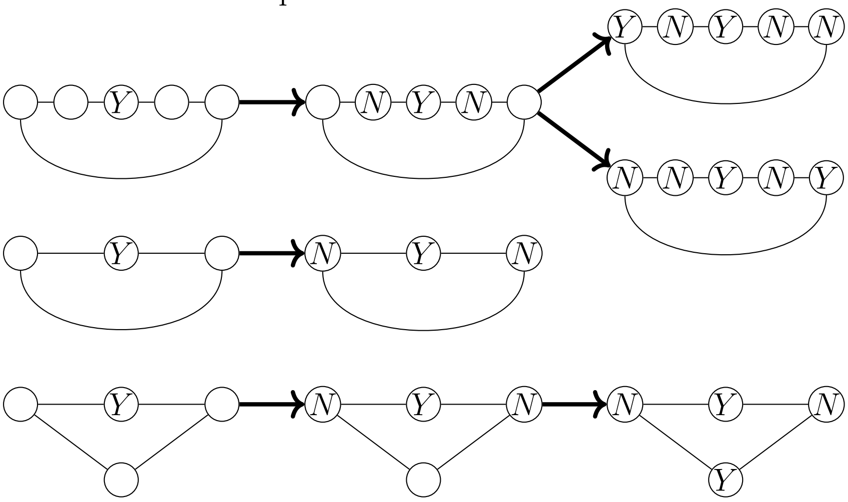
  Note that when we say neighbor of v, it means the neighbor of v that hasn't made strategy. Let i goes through 1 to $2n+1$, we can find all possible Nash equilibrium by using the algorithm above. (May have some same Nash equilibrium)

- When G is an even cycle. Denote the vertices of G: $v_1, v_2, \cdots, v_{2n}$, where $n \geq 2$. Let v_i's strategy be Y. We follow this algorithm to construct the Nash equilibrium:

  1. Let v_n, v_m be neighbors of v_i. Since v_i chose Y, v_n, v_m must choose N.
  2. Let v_x be neighbor of v_m, v_y be neighbor of v_n. If v_x, v_y are same point, then choose Y, algorithm ends. Otherwise, both of them could choose Y, then perform step 3.
  3. Let v_p be neighbor of v_x, v_q be neighbor of v_y. If v_p, v_q are same point, then choose N, algorithm ends. Otherwise, both of them could choose N, then let $v_m = v_p, v_n = v_q$, perform step 2.

  Note that when we say neighbor of v, it means the neighbor of v that hasn't made strategy. Let i goes through 1 to $2n$, we can find all possible Nash equilibrium by using the algorithm above. (May have some same Nash equilibrium)

Here are some examples:



* Note that any pure strategy profile on a cycle has no two consecutive vertices play Y, ad no three consecutive vertices play N is a NE.

2.2 General Case

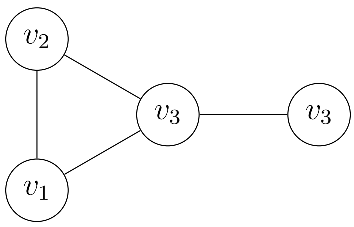
- Denote the probability of the vertex v_i plays Y as p_i, as a pure NE, $p_i \in \{0, 1\}$.
- We first find the best response function for $v_i \in V(G)$:

$$B_{v_i} = (p_i, 1 - p_i), \text{ where } p_i = \prod (1 - p_j), \forall e_{i,j} \in E(G)$$

- By listing all $v_i \in V(G)$, we can obtain an equations-system, solving gives us the pure NE as desired.

$$\forall v_i \in V(G), \ p_i = \prod (1 - p_j), \forall e_{i,j} \in E(G)$$

- Example: consider the graph



Now we have the equation-system:

$$\begin{aligned} p_1 &= (1 - p_2)(1 - p_3) \\ p_2 &= (1 - p_1)(1 - p_2) \\ p_3 &= (1 - p_1)(1 - p_2)(1 - p_3) \\ p_4 &= (1 - p_3) \\ p_1&, p_2, p_3, p_4 \in \{0, 1\} \end{aligned}$$

It gives us the pure NE: each of $\{v_1, v_4\}, \{v_2, v_4\}, \{v_3\}$ are the vertices playing Y.

* Note that this is also a way to find the independent dominating set in a graph.

3 Real Life Applications

Here's the parking lots near university of Waterloo by Oct 8, 2024.

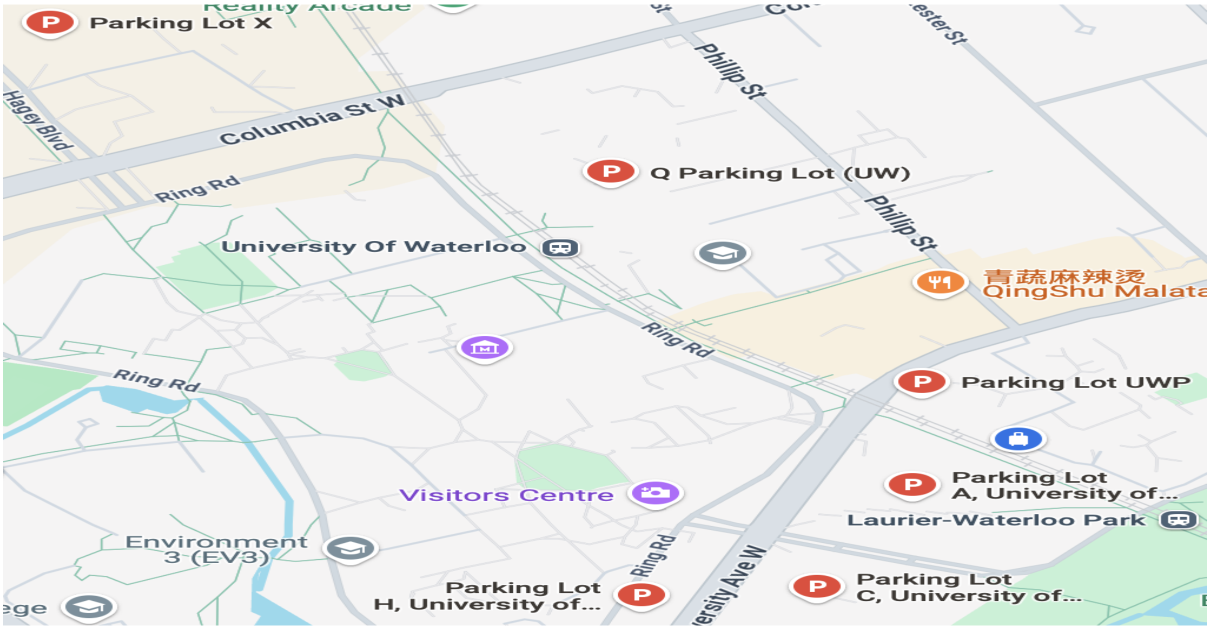
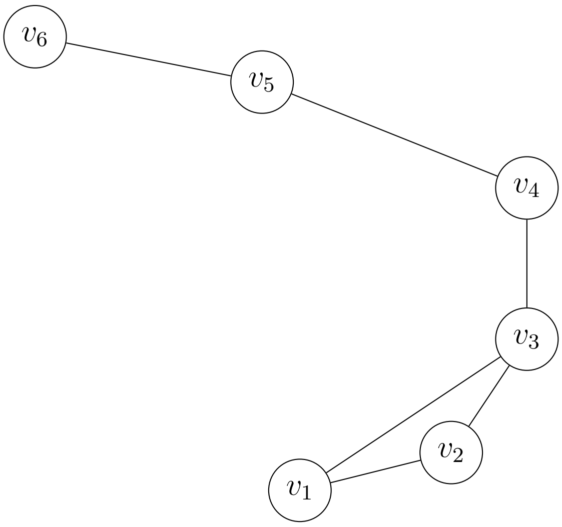


Figure 1: Parking lots

We can derive the parking lots into a graph.



The equation-system is given by:

$$
\begin{aligned}
p_1 &= (1 - p_2)(1 - p_3) \\
p_2 &= (1 - p_1)(1 - p_3) \\
p_3 &= (1 - p_1)(1 - p_2)(1 - p_4) \\
p_4 &= (1 - p_3)(1 - p_5) \\
p_5 &= (1 - p_4)(1 - p_6) \\
p_6 &= (1 - p_5) \\
& p_1, p_2, p_3, p_4, p_5, p_6 \in \{0, 1\}
\end{aligned}
$$

One solution (NE) is $\{v_1, v_4, v_6\}$ plays Y, $\{v_2, v_3, v_5\}$ plays N.

4 Conclusion

In conclusion, the problem of deciding whether to build a charging station at a parking lot can be modeled as a strategic game on a graph where each player aims to maximize their utility. Our analysis shows that in this game, equilibrium is reached when parking lots that are sufficiently spaced apart build charging stations, while their neighbors refrain from doing so. This behavior emerges naturally from the selfish motives of each parking lot (player) to avoid competition with neighboring stations and maximize profit. Therefore, the system self-organizes into a pattern where only non-adjacent parking lots choose to build charging stations, ensuring that each station can operate profitably.

5 Real-Life Implications

This research highlights a real-life phenomenon where decentralized decision-making leads to an efficient distribution of charging stations. In urban planning, this suggests that charging stations should be strategically placed to avoid unnecessary competition, improving the overall efficiency of electric vehicle infrastructure. Moreover, understanding this game-theoretic behavior can help city planners or policymakers create incentives or regulations that encourage cooperation between neighboring parking lots, potentially leading to more optimal and widespread charging station coverage.

6 References

1. Martin Pei, "Game Theory Assignments 2". [private access]
2. University of Victoria, "A Simple Algorithm for Dominating Set". [Online] http://webhome.cs.uvic.ca/~wendym/courses/425/14/notes/425_03_dom_alg.pdf. [Accessed: 9/10/2024]
3. Wayne Goddard and Michael A. Henning, "Independent Domination in Graphs" [Online] https://people.computing.clemson.edu/~goddard/papers/idomSurvey.pdf. [Accessed: 9/10/2024]
4. Stanford University. "Game Theory" [Online] https://plato.stanford.edu/entries/game-theory/. [Accessed: 9/10/2024]
5. WIKIPEDIA, "Nash equilibrium". [Online] https://en.wikipedia.org/wiki/Nash_equilibrium. [Accessed: 9/10/2024]
6. WIKIPEDIA, "Independent set (graph theory)". [Online] https://en.wikipedia.org/wiki/Independent_set_(graph_theory). [Accessed: 9/10/2024]
7. WIKIPEDIA, "Dominating set". [Online] https://en.wikipedia.org/wiki/Dominating_set. [Accessed: 9/10/2024]
8. Google Map, "Parking lots near University of Waterloo". [Online] https://www.google.com/maps/search/parking+lot/@43.4726438,-80.5400744,15.74z?hl=en&entry=ttu&g_ep=EgoyMDIOMTAwNS4yIKXMDSoASAFQAw%3D%3D. [Accessed: 9/10/2024]