An Analysis of the Public Goods Game: Cooperation, Incentives, and Group Dynamics

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December 3rd, 2024

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1 Introduction

1.1 Background and Motivation

Writing the CO456 Final Project can be an overwhelming task during exam season. Students form groups of up to 3 students to write the project, and all members of a group will receive the same grade. It may be tempting to "let the other group members write the project" while focusing on studying for exams, maximizing one's utility (overall course grade). This creates a classic free-rider dilemma: if everyone adopts this strategy, no one works on the project, and the entire group receives a grade of 0, highlighting the risks of misaligned incentives in shared tasks — much like the Public Goods Game.

The Public Goods Game (PGG) is a foundational model in experimental economics and behavioral science, exploring the tension between individual rationality and collective welfare. Public goods problems are pervasive in real-world scenarios such as climate change mitigation, public health efforts, and tax compliance [1]. The theoretical simplicity of the PGG, coupled with its adaptability to complex real-world scenarios, makes it an invaluable tool for understanding cooperation dynamics [2].

Despite the Nash equilibrium predicting free-riding as the dominant strategy, empirical studies demonstrate that individuals often deviate from purely self-interested behavior [3]. This opens up questions about the underlying motivations for cooperative behavior and the mechanisms that can sustain it in the long run [4].

1.2 Research Objectives

This study focuses on dissecting the multifaceted dynamics of cooperation in public goods scenarios. Specifically, we aim to address:

- The behavioral and structural factors influencing contribution levels [5].
- The effectiveness of interventions such as transparency, competition, and incentive mechanisms in fostering cooperation [6][7].
- The temporal dynamics of cooperation in iterated public goods games and strategies to counter declining contributions over time [8].

1.3 Research Contributions

Our work analyzes PGG at a deeper scale and extends the PGG framework by:

- ⋆ Incorporating intergroup competition to examine its influence on cooperative behavior [9].
- ⋆ Investigating the interplay between punishment and reward mechanisms and their combined effects [4][5].
- ⋆ Using behavioral modeling to simulate alternative scenarios and predict long-term outcomes [10].

2 Theoretical Framework

2.1 The Basic Public Goods Game Model

In the standard PGG, N players each start with an initial endowment E and decide how much to contribute ($x_i \in [0, E]$) to a public pool. Contributions are multiplied by a factor $M > 1$, representing the collective benefit, and redistributed equally among all players. The individual payoff is given by:

$$\pi_i = E - x_i + \frac{M \cdot \sum_{j=1}^{N} x_j}{N}.$$

- ▷ **Dominant Strategy**: Rational players contribute nothing ($x_i = 0$). This maximizes individual payoff irrespective of others' contributions [1].
- ▷ **Social Optimum**: Full contribution ($x_i = E$) maximizes the group's collective payoff, illustrating the classic tension between individual and collective rationality [8].

2.2 Nash Equilibrium where $M < N$

The Nash equilibrium is a strategy profile where no player can unilaterally improve their payoff by changing their contribution, given the contributions of others. In the standard Public Goods Game (PGG) setup, the Nash equilibrium can be derived as follows:

- **Assumptions:** Each player i has an initial endowment E and chooses a contribution $x_i \in [0, E]$ to the public pool. The total contributions are multiplied by a factor $M > 1$ and equally distributed among N players. Recall the payoff for player i is:

$$\pi_i = E - x_i + \frac{M \cdot \sum_{j=1}^{N} x_j}{N}.$$

- **Individual Incentive:** The marginal cost of contributing is 1, while the marginal benefit of an additional unit contributed is $\frac{M}{N}$. For this case, assume $M < N$, the marginal benefit is always less than the cost [1][11].
- **Dominant Strategy:** A rational, self-interested player maximizes their payoff by contributing nothing ($x_i = 0$), regardless of the contributions of others [9].
- **Equilibrium Outcome:** In NE, all players choose $x_i = 0$. The total public pool is zero, and each player's payoff is: $\pi_i = E$.

  While the Nash equilibrium represents a stable outcome where no player has an incentive to deviate unilaterally, it does not maximize social welfare. In our problem, this means all members receive mark of 0. The social optimum occurs when all players contribute their full endowment ($x_i = E$), leading to:

  - ◇ **Total Contribution:** The public pool is maximized at $N \cdot E$ [8].
  - ◇ **Payoff per Player:** Each player's payoff becomes:

$$\pi_i = \frac{M \cdot N \cdot E}{N} = M \cdot E.$$

  Since $M > 1$, this outcome is strictly better for every player compared to the Nash equilibrium payoff of E [4].

The conflict arises because contributing to the public pool incurs an individual cost, while the benefits are shared among all players. This misalignment of individual and group incentives leads to the *tragedy of the commons*, where individual rationality leads to suboptimal outcomes for the group [11].

This happens when members are more likely to free-ride because their individual effort on the project does not significantly increase the overall grade. They might focus on other priorities (e.g., studying for exams) since contributing feels less rewarding.

2.3 Nash Equilibrium with M $\geq N$

When $M \geq N$, the dynamics of the Public Goods Game (PGG) change significantly because the marginal benefit of contributing to the public good exceeds or equals the marginal cost. In this scenario:

- **Assumptions:** Similar to the $M < N$ case, each player i has an initial endowment E and chooses a contribution $x_i \in [0, E]$. The payoff formula for player i remains the same:
$$\pi_i = E - x_i + \frac{M \cdot \sum_{j=1}^{N} x_j}{N}.$$
[8]
- **Individual Incentive:** The marginal cost of contributing remains 1, while the marginal benefit of an additional unit contributed is $\frac{M}{N}$. When $M \geq N$, the marginal benefit satisfies:
$$\frac{M}{N} \geq 1.$$
This means contributing to the public pool becomes individually rational because the benefit to the player from contributing exceeds or equals the cost [9].
- **Dominant Strategy:** In this case, the dominant strategy for each rational player is to contribute fully ($x_i = E$), as this maximizes both individual and collective payoffs [3].
- **Equilibrium Outcome:** In the Nash equilibrium, all players contribute their full endowment ($x_i = E$), leading to a total public pool of:
$$\text{Total Contribution} = N \cdot E.$$
Each player's payoff becomes:
$$\pi_i = E - E + \frac{M \cdot N \cdot E}{N} = M \cdot E.$$
This outcome coincides with the maximum social welfare as the group collectively achieves the highest possible payoff, and there is no conflict between individual and collective interests [4].
This is the case when overall project grade would improve significantly with individual contributions. For example, if the group divides the workload efficiently (e.g., one member handles research, another writes, and another reviews), each person's input clearly boosts the project quality and, thus, the shared grade.

Implications:

- ◇ **Alignment of Interests:** Unlike the $M < N$ case, where individual rationality leads to free-riding, the $M \geq N$ scenario aligns to individual and group incentives, encouraging full cooperation and maximizes social welfare.
- ◇ **No Free-Rider Problem:** Since contributing fully is in each player's best interest, the free-rider problem does not occur, and there is no need for external interventions such as rewards or punishments to sustain cooperation.
- ◇ **Real-World Applications:** Situations where $M \geq N$ can model scenarios with high efficiency multipliers, such as small teams with strong synergies or high-return public goods projects (e.g., corporate innovation or breakthrough research initiatives).

While $M \geq N$ creates an ideal environment for cooperation, real-world factors like uncertainty about others' contributions or lack of trust may still reduce participation. However, such issues are less prevalent compared to the $M < N$ case, as the incentives for cooperation are inherently stronger.

In both cases of Nash equilibrium, real-world deviations often occur due to factors such as conditional cooperation, social norms, or repeated interactions, as discussed at a greater detail in Section 2.6.

2.4 Shapely Values

- The Shapley value can be used to fairly distribute the benefits of the public good based on individual contributions [9]. It helps determine how much each student's effort contributes to the final grade, accounting for their marginal contributions when working alone or in collaboration with others. For example, if the total payoff of the group is $v(N)$, the Shapley value accounts for the marginal contributions of each player across all possible coalitions [9].
- **Example:** Consider a PGG with three players, where the total value of the public good is $v(N) = 60$. If the individual and coalition contributions are:
$$v(\{1\}) = 10, \quad v(\{2\}) = 20, \quad v(\{3\}) = 30,$$
$$v(\{1,2\}) = 40, \quad v(\{1,3\}) = 50, \quad v(\{2,3\}) = 55,$$
the Shapley value distributes the total payoff among players based on their marginal contributions to each coalition [9].
- **Fairness and Insights:** The Shapley value ensures that players who contribute more to the public good receive a higher share of the payoff, promoting fairness and incentivizing cooperation [8], [9].

The Shapley value complements the Nash equilibrium by addressing the distribution of payoffs in cooperative settings, offering a framework for fair allocation in both theoretical and practical applications [9].
From this, we can conclude that adopting a mechanism inspired by the Shapley value to the CO456 project could help fairly allocate grades or recognition among group members, discouraging free-riding and promoting cooperation. For example, peer evaluations could help measure and weight individual contributions, leading to fairer group outcomes.

2.5 The Core

In the PGG, the total value $v(N)$ is derived from the contributions of all players to the public pool, multiplied by the benefit factor M [9]. The core captures payoff allocations where:

- ◇ No group of players would prefer to form a smaller coalition and allocate the public good among themselves [4], [7].
- ◇ Each player's payoff reflects their contribution to the public pool, ensuring fairness [9], [11].

The core is non-empty in the Public Goods Game (PGG) if certain conditions are met, ensuring that no subset of players (coalition) can achieve a better outcome by breaking away from the group [5].

- ▷ *Coalitional Stability in the Public Goods Game:* $\sum_{i \in S} \pi_i \geq v(S)$,
where $v(S)$ is the total value generated by coalition S, calculated as: $v(S) = M \cdot \sum_{j \in S} x_j$.

Here:

- ▷ M: The public goods multiplier, which determines the efficiency of contributions [10].
- ▷ $\sum_{j \in S} x_j$: The total contributions made by players in coalition S [9].
- ▷ π_i: The payoff received by player i in the group allocation [4].

Coalitional stability ensures that no subset of players S has an incentive to leave the larger group and create their own public goods pool [4], [7].

- ✓ Coalitional stability depends on the fairness and efficiency of the payoff distribution [9]. For example:

$$\pi_i \propto \frac{x_i}{\sum_{j \in N} x_j} \cdot M \cdot \sum_{j \in N} x_j,$$

  where each player's share is proportional to their contribution relative to the total [9], [10].
- ✓ If a high-contributing coalition S feels underrewarded (e.g., their marginal contributions are not adequately recognized), they may form a breakaway group to maximize their collective benefit [4], [7].
- ✓ For coalitional stability, it is essential that players in any subset S receive a share of the total public good value $M \cdot \sum_{j \in N} x_j$ that is greater than or equal to their standalone value $M \cdot \sum_{j \in S} x_j$ [5], [9].
- ✓ This condition becomes more challenging as the public goods multiplier M decreases or if the allocation mechanism disproportionately benefits free-riders or low-contributing members [6], [10].

Example:

Consider a PGG with $N = 5$ players, each with an endowment of $E = 10$, and a multiplier $M = 2$ [10]. Suppose coalition $S = \{1, 2\}$ contributes $x_1 = 8$ and $x_2 = 6$, while the total contributions in the group are $\sum_{j \in N} x_j = 40$ [9].

- ⋆ The value of the coalition acting alone is:

$$v(S) = M \cdot \sum_{j \in S} x_j = 2 \cdot (8 + 6) = 28.$$

- ⋆ For coalitional stability, the payoffs for players 1 and 2 in the full group must satisfy:

$$\pi_1 + \pi_2 \geq 28.$$

- ⋆ If the allocation to players 1 and 2 in the group is less than 28, they have an incentive to leave and form their own public goods pool.

The core in this context represents a fair and stable allocation of the project grade among group members, ensuring that no subset of students (a coalition) would benefit from leaving the group to work on the project independently.

- ✓ Fair Distribution: The final grade (the "public good") should reflect each group member's contributions. If members feel their contributions are not adequately rewarded, they might prefer to form a smaller group or work alone.
- ✓ Coalitional Stability: No subgroup of students (coalition) should be able to achieve a better grade by working separately. For the group to remain stable, every student must feel they are better off collaborating than splitting away.

In our problem, the grade for all students is the same regardless of their contributions, hence the core could be empty because the condition for coalitional stability may not be satisfied. Since we cannot be certain whether working alone would yield a higher grade compared to collaborating, we cannot definitively conclude whether the core in our problem is empty or not. Note that sunk costs also have a psychological dimension. While sunk costs are not directly part of the core's theoretical framework, they can influence students' perceptions and decisions. By this point in the project, breaking out of the coalition and restarting the project may feel like it is not worth the effort, even if it might theoretically yield a better outcome.

2.6 Behavioral Extensions

While the basic Public Goods Game (PGG) assumes purely self-interested players, empirical studies show that human behavior often deviates from the Nash equilibrium [9]. These deviations are influenced by psychological, social, and contextual factors, leading to increased contributions in many real-world scenarios [6], [8]. This section explores key behavioral extensions that enrich the standard PGG framework.

- ✓ **Conditional Cooperation**: Some individuals contribute based on observed contributions from others, fostering reciprocity [3], [9].
- ✓ **Inequity Aversion**: Players may adjust their contributions to reduce perceived unfairness in payoff distributions [6], [10].
- ✓ **Altruism and Social Norms**: Intrinsic motivations and adherence to social norms can override purely rational strategies [7], [11].
- ✓ **Reputation Effects:** In settings where contributions are visible, players may contribute more to maintain a positive reputation [8], [6]. Transparency and social accountability incentivize higher contributions, as individuals seek approval or avoid social disapproval [9].
- ✓ **Time Preferences and Declining Contributions:** In repeated games, contributions often decline over time due to the following reasons:
  - ▷ *Disillusionment:* Players reduce contributions if they perceive others are free-riding [6].
  - ▷ *Endgame Effects:* Near the final rounds, players anticipate the lack of future reciprocity and shift toward free-riding [4].

  Interventions such as rewards, punishments, or reminders of social norms can mitigate this decline [5], [9].
- ✓ **Group Dynamics:** Group size and composition significantly influence contributions. Smaller groups tend to foster higher cooperation levels due to stronger social bonds and easier monitoring of free-riders [10]. Heterogeneous groups, where individuals have varying endowments or valuations, may experience more inequity-driven dynamics [7]. This is why the writer of this project chose to form a two-person group, it was a strategic decision based on our project analysis ;)
- ✓ **Punishments and Rewards:** Introducing penalties for free-riders or bonuses for high contributors can shift the equilibrium toward cooperation [5]. Behavioral studies suggest that even minor rewards or punishments can have outsized effects by reinforcing social norms and deterring selfish behavior [8].

These behavioral extensions highlight the complexity of human decision-making in public goods scenarios [9], [10]. Incorporating these factors into theoretical and experimental designs enriches our understanding of cooperation and provides actionable insights for real-world applications, such as climate action, public health campaigns, and resource management [6], [5].

2.7 Advanced Mechanisms

To enhance the realism and applicability of the PGG, the following mechanisms are introduced:

- **Transparency**: Revealing individual contributions can increase accountability and motivate higher contributions [8], [6].
- **Punishments and Rewards**: Penalizing free-riders and rewarding high contributors can shift equilibria toward more cooperative outcomes [5], [8].
- **Group Competition**: Adding intergroup dynamics introduces social comparison, encouraging groups to outperform others by fostering internal cooperation [10].

3 Methodology

3.1 Experimental Design

We employ a multi-faceted experimental approach to isolate the effects of various factors:

- **Baseline Public Goods Game**: $N = 5, E = 10, M = 1.5, 2.0, 3.0$, with both anonymous and transparent settings [4], [5].
- **Group Competition**: Groups compete under the following treatments:
  - No competition (PG) [7].
  - Competition without incentives (XPG) [8].
  - Competition with incentives (CPG) [5].
- **Iterated Games**: Over 10 rounds, temporal trends in contributions are analyzed [5], [9].

3.2 Data Collection and Metrics

Key metrics include:

⇒ Average contributions, free-riding rates, and group payoffs [5].

⇒ Behavioral trends across rounds to assess stability and decline [4], [9].

3.3 Analytical Techniques

- **Regression Analysis**: Quantifies the effects of transparency, competition, and incentives on contribution levels [6], [10].
- **Behavioral Modeling**: Simulates alternative scenarios to predict long-term dynamics under different interventions [5], [7].
- **Hypothesis Testing**: Examines the statistical significance of observed trends and deviations from theoretical predictions [8].
- **Simulation-Based Analysis**: The custom simulation code serves as a powerful tool for testing scenarios that are analytically complex. By iterating through multiple rounds of the Public Goods Game, we analyze trends in contributions, the stability of cooperation, and the effectiveness of interventions such as dynamic rewards or punishments [6]. This approach provides insights into behaviors that cannot be derived easily from closed-form mathematical models. Please see Section 6 for an example code sample.
- **Visualization and Data Exploration**: Visualization techniques, including bar charts and line plots, are employed to explore and communicate trends in contributions and payoffs [5]. These visualizations are generated directly from the simulation outputs, providing a clear view of player dynamics and public good utilization. See Section 6 for bar chart examples.
- **Time-Series Analysis**: Temporal trends in contributions are analyzed using time-series techniques to identify patterns such as declining cooperation over rounds or sudden changes due to interventions [4], [10]. This helps us understand how behaviors evolve over time and which mechanisms sustain or erode cooperation.

4 Results and Discussion

4.1 Baseline Findings

- Contributions increase with higher multipliers ($M = 3.0$) due to enhanced returns on public investment [5], [10].
- Transparent conditions foster greater cooperation than anonymous settings, highlighting the role of accountability [6], [7].

4.2 Intergroup Competition

- Without incentives, competition (XPG) primarily motivates contributions through social comparison [9], [8].
- With incentives (CPG), sustained cooperation is observed as groups aim to maximize relative performance [5], [10].

4.3 Iterated Game Dynamics

✓ Contributions decline over repeated rounds in the absence of interventions, consistent with prior studies [4], [9].

✓ Punishments and rewards, particularly when combined, counteract this decline effectively [7], [5].

5 Conclusion

We thoroughly analyzed whether to collaborate on writing this CO456 project by exploring the Public Goods Game, but since you're reading this fully finished report, it seems we've decided to stick with the coalition—no turning back now!

6 Simulation Code

As discussed in prior sections, the Public Goods Game dynamics could be simulated with Python. Below is sample code that calculates contributions and payoffs for each round under various strategies, including cooperative, defecting, and conditional cooperation. We analyze the few outcomes below.

```
import numpy as np
import matplotlib.pyplot as plt

# Parameters
N = 10  # Number of players
E = 10  # Initial endowment per player
M = 2.0  # Multiplier for public goods
rounds = 10  # Number of rounds
reward = 1.0  # Reward for contributing
punishment = -1.0  # Punishment for free-riding

# Player Strategies: 1 = Always Cooperate, 0 = Always Defect, 0.5 = Conditional Cooperator
strategies = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0]  # This is an example Of No Cooperation

def public_goods_game(N, E, M, rounds, strategies, reward=0, punishment=0):
    contributions = np.zeros((rounds, N))
    payoffs = np.zeros((rounds, N))

    for t in range(rounds):
        for i in range(N):
            if strategies[i] == 1:  # Always Cooperate
                contributions[t, i] = E
            elif strategies[i] == 0:  # Always Defect
                contributions[t, i] = 0
            else:  # Conditional Cooperator
                avg_contribution = np.mean(contributions[t - 1, :]) if t > 0 else 0
                contributions[t, i] = E if avg_contribution > E / 2 else 0

        # Total contribution and redistribution
        total_contribution = np.sum(contributions[t, :])
        public_good = total_contribution * M
        payoff_share = public_good / N

        for i in range(N):
            # Base payoff
            payoffs[t, i] = E - contributions[t, i] + payoff_share

            # Apply reward and punishment
            if contributions[t, i] > 0:
                payoffs[t, i] += reward
            elif contributions[t, i] == 0:
                payoffs[t, i] += punishment

    return contributions, payoffs

contributions, payoffs = public_goods_game(N, E, M, rounds, strategies, reward, punishment)
```

6.1 No Cooperation

Strategies: [0, 0, 0, 0, 0, 0, 0, 0, 0, 0]
This simulates a scenario where all players defect and contribute nothing to the public good [3].

- Total contribution = 0
- Public good = $M \times$ Total Contribution = 0
- Payoff for each player: $\pi_i = E +$ Punishment where Punishment $= -1$. Thus, all players end up with $\pi_i = E - 1$ [4].
- We observe uniform punishment where each player received the same penalty of -1 for defecting [5].
- Since no contributions or rewards were made, all players ended with equal payoffs [7].

See Figure 1

6.2 All Players Cooperate

Strategies: [1, 1, 1, 1, 1, 1, 1, 1, 1, 1]
This simulates a scenario where everyone contributes their entire endowment to the public good [8].

- Total contribution $= N \times E$
- Public good $= M \times$ Total Contribution
- Payoff = Equal for all players, as redistribution is proportional [9].
- This scenario yields the **maximum social outcome**, as the collective payoff (sum of all individual payoffs) is maximized. Full cooperation eliminates free-riding and ensures that the group's resources are efficiently utilized [11].

See Figure 2

6.3 Half the Players Cooperate, Half Defect

Strategies: [1, 1, 1, 1, 1, 0, 0, 0, 0, 0]
This setup splits the group into contributors and defectors [10].

- Defectors receive higher payoffs (free-riding) since they benefit from the public good without contributing [6].
- Contributors receive less than defectors because they lose their contribution =¿ Free-riders problem [12].

See Figure 3

6.4 Mixed Strategies: Cooperators, Defectors, and Conditional Cooperators

Strategies: [1, 1, 0, 0, 0, 0.5, 0.5, 0.5, 1, 0]
A realistic mix of strategies where some always cooperate, some always defect, and some conditionally cooperate [2].

- Conditional cooperators contribute when cooperators dominate [5].
- Defectors benefit the most if many players cooperate [9].
- Dynamic and interesting behavior across rounds [8].

  See Figure 4

7 Appendix (Figures)

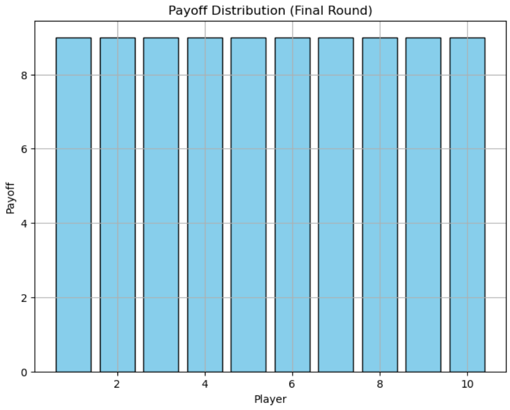

Figure 1: No Cooperation

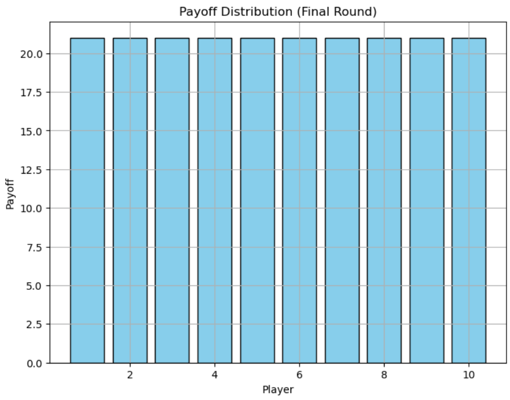

Figure 2: All Players Cooperate

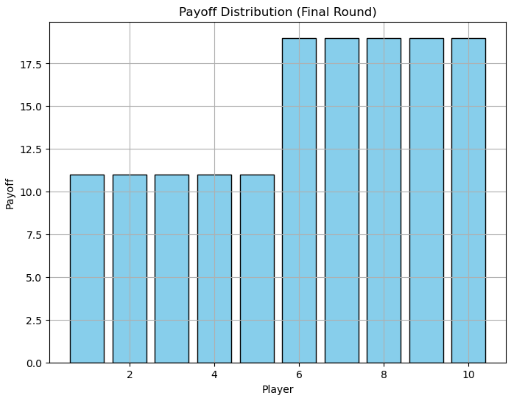

Figure 3: Half the Players Cooperate, Half Defect

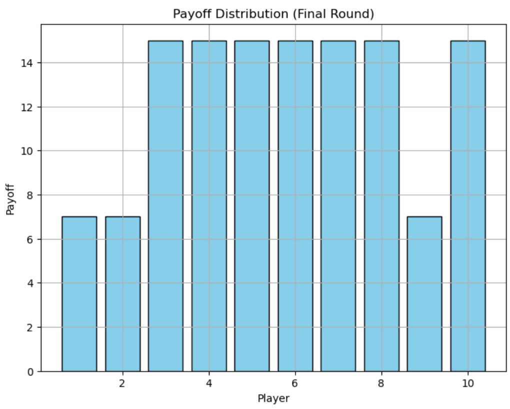

Figure 4: Mixed Strategies

8 References

1. Andreoni, J. "Cooperation in Public Goods Experiments: Kindness or Confusion?" *Economics Letters*, Volume 57, Issue 3, 2005. Available at: https://www.sciencedirect.com/science/article/abs/pii/S0167268105002532?via%3Dihub.

2. Dong, Y., Zhang, B. & Tao, Y. The dynamics of human behavior in the public goods game with institutional incentives. *Sci Rep* **6**, 28809 (2016). https://doi.org/10.1038/srep28809.

3. Fischbacher, U., Gächter, S., & Fehr, E. "Are People Conditionally Cooperative? Evidence from a Public Goods Experiment." *CESifo Working Paper Series*, No. 3187. Available at: https://www.cesifo.org/DocDL/cesifo1_wp3187.pdf.

4. Fehr, E., & Gächter, S. "Cooperation and Punishment in Public Goods Experiments." *American Economic Review*, Volume 90, Issue 4, 2000. Available at: https://doi.org/10.1257/aer.90.4.980.

5. Gächter, S., Renner, E., & Sefton, M. "The Long-Run Benefits of Punishment." *Science*, Volume 322, Issue 5907, 2008. Available at: https://doi.org/10.1126/science.1164744.

6. Henrich, J., Heine, S. J., & Norenzayan, A. "The Weirdest People in the World?" *Behavioral and Brain Sciences*, Volume 33, Issue 2-3, 2010. Available at: https://doi.org/10.1017/S0140525X0999152X.

7. Kahan, J. P. "Public Goods and Free Riders: A Cross-Disciplinary Perspective." *Archived Research Paper*. Accessed via the Wayback Machine: https://web.archive.org/web/20120328062400/http://citation.allacademic.com/meta/p_mla_apa_research_citation/0/6/4/8/2/pages64827/p64827-1.php.

8. Nowak, M. A. "Five Rules for the Evolution of Cooperation." *Science*, Volume 314, Issue 5805, 2006. Available at: https://www.science.org/doi/10.1126/science.1133755.

9. Rand, D. G., & Nowak, M. A. "Human Cooperation." *Trends in Cognitive Sciences*, Volume 17, Issue 8, 2013. Available at: https://doi.org/10.1016/j.tics.2013.06.003.

10. Santos, F. C., Santos, M. D., & Pacheco, J. M. "Social Diversity Promotes the Emergence of Cooperation in Public Goods Games." *Journal of Theoretical Biology*, Volume 267, Issue 1, 2014. Available at: https://www.sciencedirect.com/science/article/abs/pii/S0022519314004226.

11. Ostrom, E. "Collective Action and the Evolution of Social Norms." *Journal of Economic Perspectives*, Volume 14, Issue 3, 2000. Available at: https://www.aeaweb.org/articles?id=10.1257/jep.14.3.137.

12. Wang, S., & Zhang, T. "Cooperation in Public Goods Games with Network Structures." *arXiv preprint*. March 2024. Available at: https://arxiv.org/abs/2403.02740.